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"*To the solid ground*
Of Nature trusts the mind which builds for aye."—WORDSWORTH.

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THE GAY SCIENCE OF ARITHMETIC.

L'Arithmétique Amusante. Par Édouard Lucas. Pp. i.-viii. 1-266. (Paris : Gauthier-Villars et Fils, 1895.)

Traité d'Aritmétique. Par C.-A. Laisant et E. Lemoine. Pp. i.-viii. 1-174. (Paris : Gauthier-Villars et Fils, 1895.)

BY the premature death of Édouard Lucas the world lost at once a wit, a scholar, and an original mathematician. This somewhat rare combination of qualities is evident in all his work, and especially in the inimitable "Récréations Mathématiques," which, to those who are able to appreciate it, will always be one of the most entertaining of books. The interest of the problems themselves, the lucidity with which they are discussed, and the brilliant appropriateness of the author's dedications and mottoes, combine to invest the work with a charm peculiarly its own.

It will be remembered that, shortly after Lucas's death, a commission was appointed by the Mathematical Society of France, consisting of the President of the Society for the time being and MM. Delannoy, Laisant and Lemoine, for the purpose of examining and classifying the unpublished manuscripts of the deceased mathematician. Two additional volumes of the "Récréations" were found practically ready for press, and these were published by the commission in 1893 and 1894 respectively. Although they unavoidably suffer to some extent from the lack of the author's final revision (for instance, we miss the usual witty dedication in some cases), these additional volumes are well worthy to rank with their predecessors.

The first of the books now under review is edited by the same commission, and is uniform in size and style with the "Récréations." It is, indeed, described on the title-page (presumably by the editors) as an introduction to the former work ; but this, though true in a sense, is rather misleading. It appears from the editors' preface that, so long ago as 1885, Lucas had in preparation a book to be called "l'Arithmétique amusante," and three manuscript note-books bearing this title were found among his papers. The book now published, or rather the first part of it (pp. 1-186), contains the text of these note-books ; the remainder consists of a series of four notes, which form a kind of supplement to the "Récréations," and complete the publication of all that Lucas has left relating to this subject.

By far the greater portion of the first part of the book has already appeared, nearly verbatim, in the "Récreations" ; the rest consists partly of arithmetical curiosities, such as

$$8 \times 123456789 + 9 = 987654321,$$

and partly of well-known tricks and puzzles, some of which are not even arithmetical ; as, for instance, how to convert a 3 into a 5 by a single stroke of the pen. This may be amusing, but is not arithmetic.

Mathematicians in search of novelty will turn with more interest to the notes which fill up the remainder of the volume. Note i. is the text of an address delivered by Lucas on August 4, 1885, at the prize-day of the Lycée Saint-Louis. Note ii. contains two interesting modifications of the problem of the jealous husbands and their wives. Note iii. gives a brief account of the scientific games invented by Lucas. Note iv., which is the most important, comprises different fragments relating to the "Récréations." These are six in number, and deal with the problem of the eight queens, games with ribbons, magic squares and cubes, and the knight's tour. The last problem, in particular, is discussed at some length.

It will be seen from what has been said that the appearance of this book adds little to Lucas's reputation, although, of course, it does nothing to diminish it. In preparing it for publication, the editors have performed a pious duty, for which they deserve our grateful acknowledgments ; and the work is likely to appeal successfully to two classes of readers. The happy possessors of the "Récréations" will not rest until they have procured this supplementary volume for the sake of the notes, or for the æsthetic purpose of completing their set ; while those who are not so fortunate will be able to make the acquaintance of the author in a comparatively easy and inexpensive way. It is only fair to warn them that, in all probability, they will feel constrained to buy the larger work afterwards.

The treatise of MM. Laisant and Lemoine is intended to serve a double purpose: to provide a strictly scientific introduction to the science of arithmetic, and to exemplify a system of orthography which has been adopted by the "Société filologique française." This is not the place to discuss the system of spelling which is advocated: suffice it to say that it is one of those compromising systems which, while rebelling against tradition, stop short, by a long way, of a strictly phonetic orthography. The consequence is a mass of inconsistency which has not even the excuse of a historical development: thus, for instance, "philosophie" becomes "filosofie," while "science" retains its traditional form. Again, "commun" is changed into "comun," while "irréductible" is left alone.

From a mathematical point of view, the book, as might be expected from the reputation of its authors, is very interesting and valuable. A treatise on elementary arithmetic may be criticised in two different ways, according as it is estimated in relation to pure science or to pedagogy. Most text-books on arithmetic are utterly unscientific, and a treatise like this, which aims at a rigorous method and, on the whole, achieves it, is a welcome acquisition.

The book deals with the four fundamental operations as applied to whole numbers and fractions; the metric system; elementary theory of numbers (prime factors, G.C.M. and L.C.M., recurring decimals, &c.); incommensurables, squares and square roots; ratio and proportion. Most of it is quite admirable; and the criticisms which follow are offered in no captious spirit, but as a kind of acknowledgment of the really scientific character of the book.

The authors begin by defining addition as an operation which is independent of the order of the things added; or rather they refrain from giving a definition of addition, and state that any definition must be subject to the condition above stated. Now in arithmetic it is not things, but numbers, that are added, and it is quite possible to give a satisfactory definition of an arithmetical sum. Thus take two groups of objects (in the most abstract sense), count the first group, then the second, and thirdly count the first group as before, but *go on counting* as you pass on to the second group instead of beginning again. Three numbers are thus obtained, and the third is defined to be the result of adding the second to the first. The commutative law follows easily enough.

The authors' definition of a sum is equally applicable to the addition of vectors, and this fact vitiates their statement that "a quantity A is said to be greater than another quantity B when A results from the addition of a quantity C to B."

The objection that concrete "quantities" are introduced, whereas pure arithmetic is concerned with numbers, and numbers only, applies to other parts of the book, notably to the chapter on fractions. It is quite true that concrete illustrations, such as those afforded by a two-foot rule, are very useful, and indeed indispensable for the purposes of primary instruction in the subject, but the *theory* of fractions is independent of these applications. This may be seen, for instance, in Biermann's "Theorie der analytischen Functionen" (after Weierstrass); and it is not difficult to see in Euclid's arithmetical books some foreshadowing of this way of looking at the matter.

Then, again, the treatment of irrational numbers, although greatly superior to that usually found in text-books, does not seem wholly satisfactory. The authors evidently intend to adopt the method of Dedekind, or rather, perhaps, that of Heine, but the way in which this is presented is not very clear. According to Dedekind, the existence of a single definite irrational or transcendental number is established when we are able to define a "Schnitt" in the (discrete) multiplicity of rational numbers; that is to say, when we are able to find a criterion which separates *all* rational numbers into two groups, A and B, such that every number, say a, which belongs to A, is greater than every number b which belongs to B. Thus, for instance, if we assign a rational number to A when its square exceeds 2, and to B when its square is less than 2, we establish a "Schnitt" which defines the irrational number $\sqrt{2}$. Heine's method is not very different from Dedekind's; thus his way of defining $\sqrt{2}$ consists in selecting from the groups A and B, as above defined, two sets of rational numbers:—

$$\begin{array}{l} a_1,\ a_2,\ a_3\ .\ .\ .\ a_n\ .\ .\ . \\ b_1,\ b_2,\ b_3\ .\ .\ .\ b_n\ .\ .\ . \end{array}$$

such that

$$\begin{array}{l} a_1 > a_2 > a_3 > .\ .\ . > a_n > .\ .\ .\ (a_n^2 > 2) \\ b_1 < b_2 < b_3 < .\ .\ . < b_n < .\ .\ .\ b_n^2 < 2) \end{array}$$

and then showing that if n, m are taken large enough, $a_n - b_m$ can be made as small as we please. MM. Laisant and Lemoine, after introducing the problem of measuring an incommensurable quantity, proceed: 'Jusqu' à présent on n'a rien trouvé de mieus pour remplir ce but que d'indiquer tous les nombres entiers ou fractionaires mesurant les quantités plus grandes et les quantités plus petites que A." Now, even if we waive the objection already brought forward, that the measurement of *quantities* is independent of pure arithmetic, and that the assumption that every quantity admits of arithmetical measurement in terms of an arbitrary unit requires justification (and this can only be given *after* the theory of irrational numbers has been established, if indeed then), the above statement is not satisfactory; because if A is really a concrete quantity, the "nombres entiers ou fractionaires," &c., can only mean "quantities commensurable with an assumed unit," and it remains to be proved that the choice of a unit has no influence on the result; while if A is a number, no criterion is given by which we can decide whether a given rational number is greater or less than A. On this point we cannot do better than refer to the preface to Dedekind's invaluable tract, "Was sind und was sollen die Zahlen?" in which the author expressly rejects all theories of irrational numbers based on the assumption of measurable quantities. That he is right in so doing must be admitted by all who reflect on the subject with sufficient attention.

These observations, as already remarked, are not intended to detract from the undoubted merit of the book. Arithmetic is a thorny subject, the very elements of which abound in points of great difficulty and delicacy; and any serious work on the science is certain to contain passages giving occasion for criticism or controversy. MM. Laisant and Lemoine deserve our gratitude for having written an original and stimulating book, which should be studied, not only by professed mathematicians, but by intelligent school teachers. Such a work ought to do much towards replacing the soul-destroying routine of ordinary school arithmetic by something of a really educative character.

In conclusion, we would ask the reader to turn to the chapter on the metric system, which contains in the compass of about a dozen pages all the "commercial arithmetic" which a French schoolboy has to learn. It seems impossible that any one, after reading this chapter, can refrain from asking himself how it is that the English nation persists in refusing to adopt a system which is as easy to learn as it is convenient to use, and which, as the experience of France has sufficiently shown, could be prescribed by the State without the risk of any, save the most transient, disturbance of trade or exchange. But we are a practical people, and the metric system was invented by "theoreticians"; therefore, we suppose, nails and perches and pennyweights will be with us to the end of time.

G. B. M.

THE STRUCTURE AND LIFE OF BIRDS.

The Structure and Life of Birds. By F. W. Headley, M.A., F.Z.S., Assistant Master at Haileybury College. 8vo. Pp. xx. + 412, with seventy-eight illustrations. (London: Macmillan and Co., 1895.)

THE author in his preface intimates that "the aim of this book is an ambitious one," namely, "to give good evidence of the development of birds from reptilian ancestors. . . . and [among other subjects] to make clear the main principles of their noble accomplishment, flight." To the former of these aims Mr. Headley devotes his first five chapters, comparing the differences and resemblances observable in the skeleton and the internal structure of birds and reptiles, and the "ancestral peculiarities," that, having "survived all change of habit," mark their relationship. He then discusses the processes of life that go on within the bird, and make it so different from its lethargic reptilian ancestors, giving a description of the anatomical structure and physiological action of its chief organs. The following paragraph, from the description of the heart and circulation, will exemplify Mr. Headley's style and method of exposition:—

> "The heart is a force-pump, which drives the blood to all parts of the body, and when it returns impure and loaded with used-up material, sends it to the lungs to be purified, after which it is despatched all over the body again. On the voyage much of it passes through the kidneys, which help the lungs to purge it of the waste of the tissues. The essentials of an efficient heart are that it should be strong, and that it should keep the pure blood separate from the impure. These two essentials are found combined in the hearts of mammals and birds. They are strong muscles; that part at least of them which forces the blood through the arteries is remarkable for its strong thick walls. And, thanks to the perfection of the machinery, the blood which has been purified in the lungs is never mixed with the impure blood which is coming from the body. The heart is divided into right and left chambers by a division through which there are no doorways. The right and left chambers are each divided into two, but there are openings from the upper into the lower, which may be closed by valves. The two lower chambers are called ventricles, and the two upper ones auricles."

A long chapter is next given to flight, one of the main subjects of the book; the machinery by which flight is accomplished, and the principles underlying the action of its complicated mechanism, with the extraneous aids the bird avails itself of to effect that purpose. Towards a solution of this difficult subject, on which, as the author remarks, "however much is learnt, a great deal more remains to be learned," he has contributed some original observations and experiments, for which we refer the reader to the book itself. A condensed but clear account of the bird's embryology and its subsequent life-history occupies several succeeding chapters. After this the book deals principally with colour and song, instinct and reason, migration and the principles of classification.

There is one statement made by the author, in speaking of the pneumaticity of the bird's bones, to which we feel inclined to take exception. Mr. Headley says on p. 107: "The hornbills, which according to good observers are very poor flyers, are as pneumatic [as to their bones] as any birds, or, perhaps, more so." The present writer's experience of hornbills in their native state extends to many species of several genera, among them the largest and heaviest of them all, the great solid-headed *Buceros galeatus*. Many of these birds, which may be seen—and the rush of their expansive wings can be heard long before that—constantly travelling at a great speed high over the tall virgin forests, and be watched by eye and ear for miles after they have passed over the observer, can scarcely, in his estimation, be accurately designated "poor flyers." They may not take such extended flights as birds on migration do, but they make journeys of considerable length; while in the tops of the highest trees, in quest of their food, they are quite as active and nimbly expert as those excellent flyers, the pigeons, which share with them their raids on the giant Urostigmas.

The volume before us presents little to take exception to, and it will be generally conceded that Mr. Headley has succeeded well in the aims he set before himself. He has produced a very instructive and thoroughly scientific book expressed in popular language, and one that will undoubtedly "prove useful," as he hopes it may, "to lovers of birds"; and, from the clear and concise manner in which the story of the bird's pedigree and life-history is narrated, it will be specially welcome to those of them who have not had the advantage of a scientific training in the subject; while it can scarcely fail to attract many of its readers, on the outlook for a spare-time occupation, to the study of ornithology, on which it will start them with a trustworthy groundwork.

OUR BOOK SHELF.

Fern Growing. Fifty years' Experience in Crossing and Cultivation, with a List of the most Important Varieties, and a History of the Discovery of Multiple Parentage, &c. By E. J. Lowe, F.R.S., F.L.S., &c. (London: John C. Nimmo, 1895.)

MR. LOWE'S name is so well known in connection with the production of hybrid ferns, that his book will be opened with interest by all those to whom the cultivation of these plants in any way appeals. But we must confess that the book falls somewhat short of our expectations